THE QUESTION OF METAL EMBRITTLEMENT IN PLASTIC DEFORMATION

A. M. Kovrizhnykh

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At the present time we are well familiar with the experimental fact that the slip line slopes in metals exhibit differences between tension and compression [1]. The qualitative explanation of this fact and the quantitative determination of the plastic deformations that arise in this case are made possible by the positions of the theory [2] which makes provision for the different resistances of the materials to plastic deformation in tension and compression. The Coulomb–Mohr plasticity condition is used in [2] for metals in the region of tensile stresses, and in the presence of an additional characteristic of the material of the internal friction angle this condition makes it possible to establish exact agreement between the yield points in uniaxial tension and torsion. It has also been demonstrated in [2] that the growth of plastic deformation leads to an increase in the internal friction angle, i.e., the embrittlement of the metal. The traditional methods of solving statically determining problems [3, 4] are based on the Trask–Saint Venant plasticity condition.

In the present study, for purposes of solving a certain class of such problems, we suggest the utilization of the Coulomb–Mohr plasticity condition for metals, which had earlier been used in the statics of free-flowing materials [5]. From this particular viewpoint, we examine the problems of plane deformation of a rigidly plastic medium: the tension and compression of a strip that has been weakened through notching, and the drawing of this strip through a short die.

1. We will denote the principal normal stresses σ_i, $i = 1, 2, 3$, and the principal axes of the stress tensor will be numbered 1, 2, 3, so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The Coulomb–Mohr plasticity condition can then be written in the following form:

$$T/\cos\varphi + \sigma \operatorname{tg}\varphi = k, \tag{1.1}$$

where $T = (\sigma_1 - \sigma_3)/2$; $\sigma = (\sigma_1 + \sigma_3)/2$; φ is the internal friction angle; k is the plastic constant. The planes on which condition (1.1) is achieved are referred to as the slip planes and these pass through the second main direction and together with the first form the angles $\pm(\pi/4 + \varphi/2)$ [5]. The internal friction angle reflects the influence of normal stress on the limit value of the tangential stress on these planes. The plastic constant $k = 0.5(1 + \sin\varphi)\sigma_s/\cos\varphi$ (σ_s is the yield point in the case of uniaxial tension).

If τ_s represents the twisting yield point, then from (1.1) we have $\sin\varphi = 0.5\cdot\sigma_s/\tau_s - 1$. For the Trask–Saint Venant plasticity condition $\varphi = 0$. From the Mises plasticity condition $\sigma_s/\tau_s = \sqrt{3}$, so that for those metals whose onset of plasticity is better described by the Mises condition than by the Trask condition we should assume $\sin\varphi = 0.15$ ($\varphi = 9°$).

We know that the associative law of flow [6, 7] leads to an irreversible change in material volume, i.e., dilatancy. However, this fact, characteristic of most rocks and free-flowing materials is not as significant in the one-time loading of plastic metals whose change in volume proceeds elastically. In this connection, in [2] we find proposed a theory of plasticity which independently provides for the effect of internal friction and dilatancy, said theory based on additional experimental data. This circumstance allows us to use the Coulomb–Mohr plasticity condition (1.1) for plastic incompressible metals.

Let us study the plane deformation of an ideal plastic medium within the scope of a rigidly plastic scheme [3, 4]. We will limit ourselves to the determination of the stress field in problems dealing with the tension and compression of a notched strip and the drawing of that strip through a short die.

Let us examine an arbitrary orthogonal coordinate system x, y, z (Fig. 1). The z axis is the principal axis and coincides with the direction 2. In the x, y coordinate system we have

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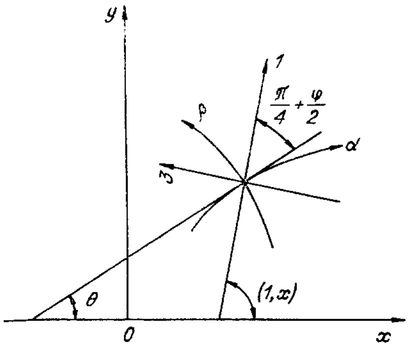

Fig. 1

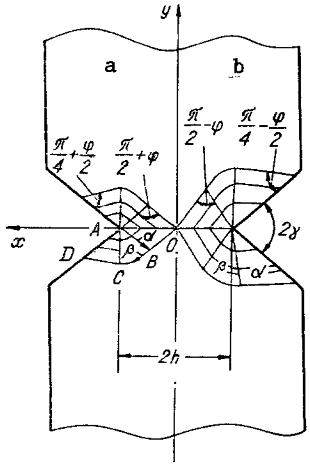

Fig. 2

$$\sigma_x = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2(1, x), \qquad (1.2)$$
$$\sigma_y = \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2}\cos 2(1, x), \quad \tau_{xy} = \frac{\sigma_1-\sigma_3}{2}\sin 2(1, x)$$

[(1, x) is the angle which forms the first principal direction with the x axis].

The line each of whose points is tangent to the slippage plane [5] is known as the slippage line in the x, y plane. It is obvious that we are dealing with two families of such lines: α and β (α is deflected to the right of the first principal direction through the angle $\pi/4 + \varphi/2$, β is deflected to the left, through the same angle).

We will denote the angle formed by the α line with the x axis as θ, since, as can be seen from Fig. 1, $(1, x) = \pi/4 + \varphi/2 + \theta$. With consideration of this substitution from (1.2), it follows that

$$\sigma_x = \sigma - T\sin(2\theta + \varphi), \qquad (1.3)$$
$$\sigma_y = \sigma + T\sin(2\theta + \varphi), \ \tau_{xy} = T\cos(2\theta + \varphi).$$

It is primarily the equations of plane equilibrium that serve as the main equations for the determination of the stress field in the plastic zone:

$$\partial\sigma_x/\partial x + \partial\tau_{xy}/\partial y = 0, \qquad (1.4)$$
$$\partial\tau_{xy}/\partial x + \partial\sigma_y/\partial y = 0,$$

which together with (1.1) and the boundary conditions in the stresses make up the statically determining problem [5].

Having substituted (1.3) into (1.4), we obtain

$$(1 + \sin\varphi\sin(2\theta+\varphi))\partial\sigma/\partial x - 2T\cos(2\theta + \varphi)\partial\theta/\partial x - \sin\varphi\cos(2\theta+\varphi)\partial\sigma/\partial y - 2T\sin(2\theta + \varphi)\partial\theta/\partial y = 0, \qquad (1.5)$$
$$-\sin\varphi\cos(2\theta+\varphi)\partial\sigma/\partial x - 2T\sin(2\theta+\varphi)\partial\theta/\partial x + (1 - \sin\varphi\sin(2\theta+\varphi))\partial\sigma/\partial y + 2T\cos(2\theta+\varphi)\partial\theta/\partial y = 0$$
$$(T = (\sigma_1 - \sigma_3)/2 = k\cos\varphi - \sigma\sin\varphi).$$

System of equations (1.5) is hyperbolic, and its characteristics coincide with the slippage lines α, β. The differential equations of the α, β family are, respectively, equal to

$$dy/dx = \mathrm{tg}\theta, \ dy/dx = -\mathrm{ctg}(\theta + \varphi). \qquad (1.6)$$

The equilibrium equations can be replaced by the relationships prevailing on the characteristics initially derived by Kётter in 1903:

$$\begin{aligned} &\operatorname{ctg}\varphi \ln(1-(\sigma/k)\operatorname{tg}\varphi) + 2\theta = \xi \text{ along the } \alpha\text{- line} \\ &\operatorname{ctg}\varphi \ln(1-(\sigma/k)\operatorname{tg}\varphi) - 2\theta = \eta \text{ along the } \beta\text{- line} \end{aligned} \tag{1.7}$$

2. Let us examine the tension (compression) problem of a strip with angled notches under conditions of plane deformation. In the formulation traditional for metals ($\varphi = 0$) the solution can be found, for example, in [3, 4], and in this case, whether for tension or for compression, the field of the slippage lines is identical. The maximum load P is calculated from the formula

$$P/(2h\sigma_s) = q/\sigma_s = 1 + \pi/2 - \gamma.$$

If $\varphi > 0$, the plastic zone and the geometry of the slippage lines for the tension of the strip or for its compression will be different. Figure 2a, b shows grids of the slippage lines for tension and compression, respectively, of a strip with angular notches.

In the y direction (Fig. 2a) let there be tension of the strip with angle notches. Since the lateral notches are free of stresses, we find from the boundary condition at AD that $\theta = -\gamma - \pi/4 - \varphi/2$, $\sigma = k\cos\varphi/(1 + \sin\varphi)$. At AO we have $\theta = -3\pi/4 - \varphi/2$, $\sigma_y = q^+ = k\cos\varphi + \sigma^+(1 - \sin\varphi)$. Using relationship (1.7) along the β line of OBCD, we now obtain

$$\begin{aligned} &\operatorname{ctg}\varphi \ln\left(1 - \frac{\sigma^+}{k}\operatorname{tg}\varphi\right) + 2\left(\frac{3}{4}\pi + \frac{\varphi}{2}\right) = \\ &= \operatorname{ctg}\varphi \ln\left(1 - \frac{\cos\varphi}{1+\sin\varphi}\operatorname{tg}\varphi\right) + 2\left(\gamma + \frac{\pi}{4} + \frac{\varphi}{2}\right), \end{aligned}$$

from which it follows that

$$\frac{q^+}{\sigma_s} = \frac{1+\sin\varphi}{2\sin\varphi}\left(1 - \frac{1-\sin\varphi}{1+\sin\varphi}\, e^{-(\pi-2\gamma)\operatorname{tg}\varphi}\right). \tag{2.1}$$

The maximum load P^+ for the tension of the notched strip is determined from the formula $P^+ = 2q^+h$.

In the case of strip compression (see Fig. 2b), without dwelling on the easily accomplished calculations, we come to the following value for the maximum load $P^- = 2q^-h$:

$$\frac{q^-}{\sigma_c} = \frac{1-\sin\varphi}{2\sin\varphi}\left(\frac{1+\sin\varphi}{1-\sin\varphi}\, e^{(\pi-2\gamma)\operatorname{tg}\varphi} - 1\right) \tag{2.2}$$

(σ_c is the yield point with uniaxial compression).

Table 1 shows the results from the calculations of the limit loads on the basis of formulas (2.1) and (2.2) for materials with different internal-friction angles, both for tension and compression of a strip with angles of $\gamma = 0$ and 30°. It follows from an analysis of these results that the internal-friction angle significantly affects the magnitude of the limit load, both in tension of the strip and in its compression.

3. Of considerable interest for the processes of pressure treatment of metals is that class of problems in which the stresses and velocities at each point in the x, y flow plane undergo no change over time, while the plastic flow is steady. Let us examine one such problem. We will turn to the study of the stress field in a rigidly plastic strip as it is

TABLE 1

φ	q^+/σ_s	q^-/σ_c	q^+/σ_s	q^-/σ_c
	$\gamma=0$		$\gamma=30°$	
0	2,57	2,57	2,05	2,05
5°	2,26	2,97	1,88	2,25
10°	2,01	3,50	1,73	2,51
15°	1,82	4,21	1,62	2,83
20°	1,66	5,19	1,51	3,24

drawn between nonmoving smooth walls of a die, these walls forming the angle γ with the x axis (Fig. 3). We will denote with H the initial thickness of the strip, while the final thickness will be denoted h. Let us assume that the die is of limited length, and that the plastic zone consists of the regions illustrated in Fig. 3. An analogous problem for an ideal plastic material in traditional formulation ($\varphi = 0$) has been solved in [3]. Let us formulate the boundary conditions of this problem for the upper half of the strip, making the assumption that a uniformly distributed pressure acts along the entire contact straight line AO. To simplify the solution of the problem, we will neglect the friction at the contact planes of the strip and the die. Thus, on AO for the stress vector we have only the normal component equal to $-q$. Therefore, in the triangle AOC we have $\theta = -\gamma - \varphi/2 - \pi/4$, $\sigma = \sigma^C = -q/(1 + \sin\varphi) + k\cos\varphi/(1 + \sin\varphi)$.

On the basis of relationship (1.7), along the slippage lines CD and BD we have

$$\ln\left[\left(1 - \frac{\sigma^C}{k}\operatorname{tg}\varphi\right)\Big/\left(1 - \frac{\sigma^B}{k}\operatorname{tg}\varphi\right)\right] = 2\operatorname{tg}\varphi\,(2\alpha + \gamma).$$

On the other hand, using (1.7) on the lines CF and BF, we write

$$\ln\left[\left(1 - \frac{\sigma^C}{k}\operatorname{tg}\varphi\right)\Big/\left(1 - \frac{\sigma^B}{k}\operatorname{tg}\varphi\right)\right] = 2\operatorname{tg}\varphi\,(2\psi - \gamma).$$

It follows from the two found relationships that $\psi = \alpha + \gamma$. An analogous relationship between the angles ψ, α, and γ is found in the familiar solution of R. Hill et al. [3] for metals without internal friction.

We can easily see in Fig. 3 that the maximum degree of deformation $\varepsilon = (H - h)/H$ is achieved when $\alpha = 0$. Then, the point C merges with D, and point B merges with F. Let us examine this particular case in greater detail, with the solution of this problem solved analytically (Fig. 4). On OB

$$\begin{gathered}\theta = -\pi/4 - \varphi/2,\ \sigma = \sigma^B,\\ \sigma_x = \sigma^B(1 - \sin\varphi) + k\cos\varphi,\\ \sigma_y = \sigma^B(1 + \sin\varphi) - k\cos\varphi,\ \tau_{xy} = 0.\end{gathered} \tag{3.1}$$

Using relationship (1.7) along the α line of ADB, we obtain

$$\sigma^B = k\operatorname{ctg}\varphi(1 - (1 - (\sigma^C/k)\operatorname{tg}\varphi)\,e^{-2\gamma\operatorname{tg}\varphi}). \tag{3.2}$$

Having determined σ^C from the boundary condition on AO in terms of q and having made the substitution into (3.2), in combination with (3.1) as a result we can find the horizontal force component on AO, equal to $p = q(H - h)$. From this we have

$$\frac{q}{\sigma_s} = \frac{(1 + \sin\varphi)\left((1 + \sin\varphi + \operatorname{ctg}\varphi)(1 + \sin\varphi)\,e^{2\gamma\operatorname{tg}\varphi} - \operatorname{ctg}\varphi\right)}{2\cos\varphi\,(2\sin\gamma\,(1 + \sin\varphi)\,e^{\gamma\operatorname{tg}\varphi} + \cos\varphi)}. \tag{3.3}$$

In the particular case under consideration, it becomes clear from the geometry of the slippage lines that

$$(H - h)/h = 2\sin\gamma\operatorname{ctg}(\pi/4 + \varphi/2)/e^{\gamma\operatorname{tg}\varphi}. \tag{3.4}$$

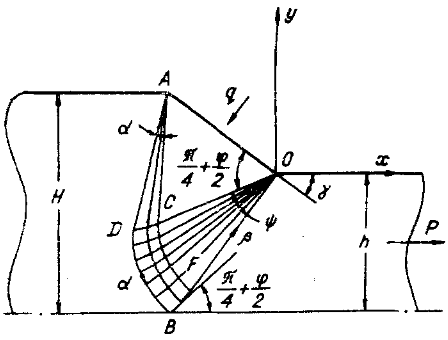

Fig. 3

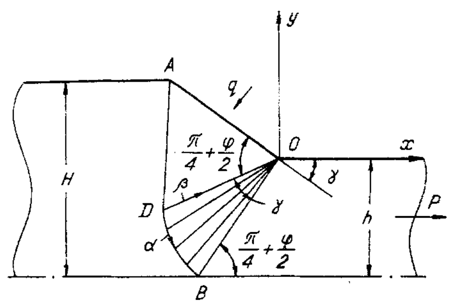

Fig. 4

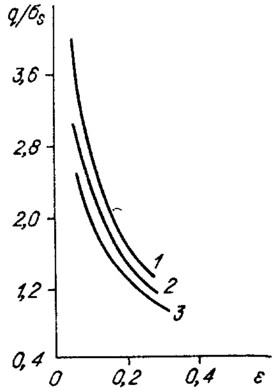

Fig. 5

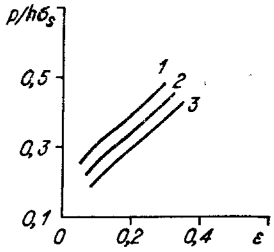

Fig. 6

In order for the strip drawing process to be stable and without any discontinuity in the right-hand side, it is necessary to satisfy the condition $p < \sigma_s h$. From this, in conjunction with the utilization of (3.3) and (3.4), we find that the drawing process is realizable when

$$\sin\gamma < \frac{\cos\varphi e^{\gamma\,\mathrm{tg}\,\varphi}}{((1+\sin\varphi)^2+(1-\sin\varphi)\,\mathrm{ctg}\,\varphi)\,e^{2\gamma\,\mathrm{tg}\,\varphi}-\mathrm{ctg}\,\varphi}. \quad (3.5)$$

If $\varphi = 0$, it then follows from (3.5) that the drawing of the strip is possible at angles of $\gamma < \gamma_* = 42°$ [3, 4]. With a change in the internal friction angle the limit angle γ_* also changes, this latter angle having been determined from inequality (3.5), as well as by the maximum degree of deformation ε_*, obtained on the basis of (3.4). As an illustration we will present the values of γ_* and ε_* for three internal-friction angles:

$$\begin{aligned} &\varphi = 0, && \gamma_* = 42°, && \varepsilon_* = 0.57,\\ &\varphi = 10°, && \gamma_* = 36°, && \varepsilon_* = 0.47,\\ &\varphi = 20°, && \gamma_* = 28°, && \varepsilon_* = 0.36. \end{aligned} \quad (3.6)$$

We can see from an analysis of the results for (3.6) that the range of angles γ for which the drawing process is possible becomes narrowed as the angle φ increases. In this case, ε_* also diminishes. We can thus draw the conclusion that the growth in the internal-friction angle leading to the embrittlement of the metal in the case of plastic flow impairs the conditions and parameters of the strip drawing process through a short die.

Let us now turn to an examination of the overall problem in which the plastic zone consists of the regions shown in Fig. 3. In this case, the solution is constructed numerically on a computer. The method used here is based on the transition from the differential relationships (1.6) and (1.7) to the finite-difference relationships and consideration of some of the properties of the slippage line (in general form, this method has been developed by Masso in 1899) [3].

Let us divide the angle γ into $\ell - 1$ equal parts of magnitudes $\delta = \gamma/(\ell - 1)$. Without limiting generality, we will examine the problem for the angles $\alpha = (m - 1)\delta$ and $\psi = (n - 1)\delta$, since such a value of δ, attributable to the selection of ℓ, can always be chosen with a sufficient degree of accuracy. It follows from the relationships derived earlier for α, ψ, and γ that $n = m + \ell - 1$. Let the subscript i be constant along the β line, and let j be constant along the α line, so that $i = 1, \ldots, n$; $j = 1, \ldots, m$. For the rectangle CDFB we have an initial characteristic problem. At the point at which the α and β lines intersect, we have $\theta_{ij} = -\pi/4 - \varphi/2 - \gamma + (i - j)\delta$. Having substituted the differential relationships (1.6) by the difference relationships, and assuming the angle θ to be equal to its average value, at the starting and ending points we can construct a grid of the slippage line.

The initially unknown pressure q on AO was determined from the condition that the sum of the horizontal components of the individual forces on the line OFB was equal to the drawing force $p = q(H - h)$. In conclusion, for a specific γ we can numerically establish the relationships that exist between p and q in dependence on $\varepsilon = (H - h)/H$.

Figure 5 shows q as a function of ε for $\gamma = 15°$ and for $\varphi = 10°, 5°, 0$ (curves 1-3). The extreme right-hand point corresponds to the analytical solution (see Fig. 4), while the extreme left-hand point corresponds to the point at which the metal is initially expelled out of the left-hand side of the die [3, 4]. This critical point is found through utilization of the solution for the compression of the strip with angled notches (see Sec. 2).

Figure 6 shows p as a function of ε for $\gamma = 15°$ and for $\varphi = 10°, 5°, 0$ (curves 1-3). Once again, the extreme right-hand point corresponds to the analytical solution (see Fig. 4), while the extreme left-hand point corresponds to the point at which the metal is initially expelled from the left-hand side of the die.

It follows from the results shown in Figs. 5 and 6 that an increase in the angle of internal friction leads to an increase both in the pressure at the walls of the die and in the drawing force. We can see from a quantitative comparison of the solutions for the determined γ and for various φ with results for various γ [3, 4] that an increase in φ by 5° corresponds to a change in the drawing force and in the pressure at the wall of the die by an equal magnitude as is the case when the angle γ is replaced by one that is analogous.

In conclusion, let us note that the extensive application in the mechanics of rocks in the case of semibrittle bodies the angle of internal friction in the case of its insignificant magnitude for metals significantly affects the force parameters in the plastic deformation of many metallic structures. Experimental observations conducted on metals [1, 8] show that this effect is most significant when the tensile stresses predominate in the deformed body.

LITERATURE CITED

1. A. Nadai, Plasticity and the Destruction of Solids [Russian translation], IL, Moscow (1954).
2. A. M. Kovrizhnykh, "Plastic deformation of reinforcing materials under complex loads," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4 (1986).
3. R. Hill, The Mathematical Theory of Plasticity [Russian translation], GITTL, Moscow (1956).
4. V. V. Sokolovskii, The Theory of Plasticity [in Russian], Vysshaya Shkola, Moscow (1969).
5. V. V. Sokolovskii, Statics in a Free Flowing Medium [in Russian], Fizmatgiz, Moscow (1960).
6. D. C. Drucker and W. Prager, "Soil mechanics and plastic analysis or limit design," Q. Appl. Math., 10, No. 2 (1952).
7. V. V. Novozhilov, "Plastic disintegration," PMM, 29, No. 4 (1965).
8. S. I. Gubkin, Plastic Deformation of Metals, Vol. 2 [in Russian], Metallurgizdat, Moscow (1961).